William Shanks Logarithms calculation

January 20, 2015

From John Radford Young 1799-1885, “Essay on the computation of logarithms” published in 1831. In 1833, he was appointed Professor of Mathematics at Belfast College. This is one of his 51 books that he wrote and are available on line. In his book he started with three equations and three variables form which he created the following three equations. His book is available on line, along with William Shanks 1853 article where he mentioned Young as the source of the equations he used to calculate the logarithms.

“Note look at the last three pages for Williams Shanks work on Euler’s Constant a lesser known number with a value of .57721 etc.”

S1 = ATANH(1/31) =lN(32/30)/2 = LN(16/15)/2
S2 = ATANH(1/49) =lN(50/48)/2 = LN(25/24)/2
S3 = ATANH(1/161) =lN(162/160)/2 = LN(81/80)/2

LN(2) = 2 * (7 * S1 + 5 * S2 + 3 * S3)
LN(3) = 2 * (11 * S1 + 8 * S2 + 5 * S3)
LN(5) = 2 * (16 * S1 + 12 * S2 + 7 * S3)

William Shanks updated values to 205 digits for the logarithms on January 21, 1854. I saw this statement at the end of his document and found it a little on the jumping the gun “The foregoing values are, it is presumed, correct to the last figure inclusive.” Well they were not very correct, see after this group where he did correct his error, granted the value of e was not perfect but he did not know that. The “<u>bold and underlined</u>” numbers are the error digits for all his work.

The correct value to 210 digits of e=.
2.7182818 2845904 5235360 2874713 5266249
7757247 0936999 5957496 6967627 7240766
3035354 7594571 3821785 2516642 7427466
3919320 0305992 1817413 5966290 4357290
0334295 2605956 3073813 2328627 9434907
6323382 9880753 1952510 1901157 3834187

This error was never check because the first 137 were correct and it was assumed all to be correct. The next person to do this calculation was J. Marcus Boorman in 1884 doing 346 digits, he also did many square root values and next followed by John von Neumann, he used ENIAC no longer the man to do the calculations rather a machine with 2010 digits just like with PI.

Williams Shanks published value.

2.7182818 2845904 5235360 2874713 5266249
7757247 0936999 5957496 6967627 7240766
3035354 7594571 3821785 2516642 7427466
3919320 0305992 1817413 5966290 4357290
0334295 2605956 3073813 2328627 9434907
6323382 98807<u>**48 2070767 3049394 92+&c**</u>

The correct value to 210 digits LN(2)=.
0.6931471 8055994 5309417 2321214 5817656
8075500 1343602 5525412 0680009 4933936
2196969 4715605 8633269 9641868 7542001
4810205 7068573 3685520 2357581 3055703
2670751 6350759 6193072 7570828 3714351
9030703 8623891 6734711 2335011 5364497

Williams Shanks published value.
0.6931471 8055994 5309417 2321214 5817656
8075500 1343602 5525412 06**79523 5847083**
2754439 2266635 5206804 5602137 0371911
8226310 4298719 4582110 0448886 1731607
5101002 4259177 6434321 7424545 3493150
6323382 9880748 2070767 3049394 92+&c

The correct value to 210 digits LN(3)=.
1.0986122 8866810 9691395 2452369 2252570
4647490 5578227 4945173 4694333 6374942
9321860 8966873 6157548 1373208 8787970
0290659 5786574 2368004 2259305 1982105
2801870 7672774 1060316 2769183 3813671
7937369 8844360 9599037 4257031 6795911

Williams Shanks published value.
1.0986122 8866810 9691395 2452369 2252570
4647490 5578227 4945173 469**3570 0667031**
1626456 2261348 7915959 6453630 4663543
4230252 7148232 3776931 0688498 5615669
0906550 5814573 8582278 9682167 2037498
0000626 1111154 1362298 9315024 24+&c

The correct value to 210 digits LN(5)=.
1.6094379 1243410 0374600 7593332 2618763
9525601 3542685 1772191 2647891 4741789
8770765 7764630 1338780 9317961 0799966
3030217 1556289 9724005 2293246 7619963
3616617 4637057 2755217 9637497 1832456
5349285 6202341 5250572 7015519 3600879

Williams Shanks published value.
1.6094379 1243410 0374600 7593332 2618763
9525601 3542685 1772191 264**6780 8257554**
5759268 0738412 2078288 5798514 2982618
5124170 8082338 1773853 3644800 7430601
6314333 5570584 1878072 7874564 5612567
3804931 0408586 1451680 34635508 54+&c

The correct value to 210 digits LN(10)=.
2.3025850 9299404 5684017 9914546 8436420
7601101 4886287 7297603 3327900 9675726

0967735 2480235 9972050 8959829 8341967
7840422 8624863 3409525 4650828 0675666
6287369 0987816 8948290 7208325 5546808
4379989 4826233 1985283 9350530 8965377

Williams Shanks published value.
2.3025850 9299404 5684017 9914546 8436420
7601101 4886287 7297603 332**6304 4104637**
8513707 3005047 7285093 1400711 3354530
3350481 2381057 6355463 4093686 9182209
1415335 9829761 8312394 5299109 9105717
7784979 7747709 8399376 1744515 35+&c

The correct value to 210 digits 1/LN(10)=m=.
0.4342944 8190325 1827651 1289189 1660508
2294397 0058036 6656611 4453783 1658646
4920887 0774729 2249493 3843174 8318706
1067447 6630373 3641679 2871589 6390656
9221064 6628122 6585212 7086568 6703295
9337086 9658826 6883311 6360773 8490514

Williams Shanks published value. There are 59 digits correct as the rest of the numbers above.
0.4342944 8190325 1827651 1289189 1660508
2294397 0058036 6656611 445**4084 2952103**
2056138 9388912 2647096 6953461 1420043
3938056 4705613 4312230 2306044 2927744
1521725 4737266 8184290 1672326 4707504
5865061 2932297 5502468 1291564 99+&c

The following table is based on the above numbers for the 14 digits around the error place. By subtracting the two numbers then divide by 2. Now dividing value by the coefficient values of s1, s2 and s3 that was used originally to multiplied by; see the three equations above. It is a clear result that the problem was that the S1 value, some calculation error that there was in that value. The bottom line is that 17 years later he did correct the problem thanks to another person who also did the same calculations to 137 digits and he found the error and William did correct the problem, see the next several pages. I have checked and no term has the needed value as listed here. It does not seem to be a missing term it must be another miss calculation as in PI.

	LN(2) error	LN(3) error	LN(5) error
Correct numbers	6800094933936	46943336374942	26478914741789
Incorrect number	6795235847083	46935700667031	26467808257554
Difference	4859086853	7635707911	11106484235
Divide by 2	2429543427	3817853956	5553242118
divide term by S1 value	347077632.4	347077632.3	347077632.3
divide term by S2 value	485908685.3	477231744.4	462770176.5
divide term by S3 value	809847808.8	763570791.1	793320302.5

By August 30, 1871 William Shanks updated his work to correct his numbers due the error pointed out by Mr. Glaisher there was an error starting at the 59 digit with his 137 digit value. Here are the new values as published. Now William Shanks new values agree with 137 digits. The new values do agree with the correct numbers within the normal calculation error which is the reason to do 205 and reduce it to 200 digits and publish only 200 digits. This is what is done today when calculating PI they many of the last digits are just dropped when comparing the two independent calculations for the correct digits.

The correct value to 210 digits LN(2)=.

0.69314 71805 59945 30941 72321 21458 17656 80755 00134 36025
52541 20680 00949 33936 21969 69471 56058 63326 99641 86875
42001 48102 05706 85733 68552 02357 58130 55703 26707 51635
07596 19307 27570 82837 14351 90307 03862 38916 73471 12335
01153 64497

Williams Shanks published value, much better.

0.69314 71805 59945 30941 72321 21458 17656 80755 00134 36025
52541 20680 00949 33936 21969 69471 56058 63326 99641 86875
42001 48102 05706 85733 68552 02357 58130 55703 26707 51635
07596 19307 27570 82837 14351 90307 03862 38916 73471 12335
01**<u>018</u>** + &c

The correct value to 210 digits LN(3)=.

1.09861 22886 68109 69139 52452 36922 52570 46474 90557 82274
94517 34694 33363 74942 93218 60896 68736 15754 81373 20887
87970 02906 59578 65742 36800 42259 30519 82105 28018 70767
27741 06031 62769 18338 13671 79373 69884 43609 59903 74257
03167 95911

Williams Shanks published value, much better

1.09861 22886 68109 69139 52452 36922 52570 46474 90557 82274
94517 34694 33363 74942 93218 60896 68736 15754 81373 20887
87970 02906 59578 65742 36800 42259 30519 82105 28018 70767
27741 06031 62769 18338 13671 79373 69884 43609 59903 74257
0**<u>2948</u>** + &c

The correct value to 210 digits LN(5)=.

1.60943 79124 34100 37460 07593 33226 18763 95256 01354 26851
77219 12647 89147 41789 87707 65776 46301 33878 09317 96107
99966 30302 17155 62899 72400 52293 24676 19963 36166 17463
70572 75521 79637 49718 32456 53492 85620 23415 25057 27015
51936 00879

Williams Shanks published value, much better.

1.60943 79124 34100 37460 07593 33226 18763 95256 01354 26851
77219 12647 89147 41789 87707 65776 46301 33878 09317 96107
99966 30302 17155 62899 72400 52293 24676 19963 36166 17463
70572 75521 79637 49718 32456 53492 85620 23415 25057 27015
51**<u>474</u>** + &c

The correct value to 210 digits LN(10)=.
2.30258 50929 94045 68401 79914 54684 36420 76011 01488 62877
29760 33327 90096 75726 09677 35248 02359 97205 08959 82983
41967 78404 22862 48633 40952 54650 82806 75666 62873 69098
78168 94829 07208 32555 46808 43799 89482 62331 98528 39350
53089 65377

Williams Shanks published value, much better.
2.30258 50929 94045 68401 79914 54684 36420 76011 01488 62877
29760 33327 90096 75726 09677 35248 02359 97205 08959 82983
41967 78404 22862 48633 40952 54650 82806 75666 62873 69098
78168 94829 07208 32555 46808 43799 89482 62331 98528 39350
5**2492** + &c

The correct value to 210 digits1/LN(10)=m=.
0.43429 44819 03251 82765 11289 18916 60508 22943 97005 80366
65661 14453 78316 58646 49208 87077 47292 24949 33843 17483
18706 10674 47663 03733 64167 92871 58963 90656 92210 64662
81226 58521 27086 56867 03295 93370 86965 88266 88331 16360
77384 90514

This number must have been too much even for William Shanks ability to do the reciprocal of a 205 digit number. For me it would take 8 pages taped together side by side, because there would be a 1 followed by 410 0's. To do the division and only 17 division steps per 8 pages group and world have required 12 such page groups a total of 96 pages. That is almost the same as the 108 pages I used to calculate PI to 100 digits.

Williams Shanks published value. There were 102 digits correct, which is an improvement.
0.43429 44819 03251 82765 11289 18916 60508 22943 97005 80366
65661 14453 78316 58646 49208 87077 47292 24949 33843 17483
18**668 38440 53039 80947 79768 71211 65951 73183 60409 55627**
56816 80637 45310 65045 32572 68778 26750 65871 94503 27872
73760 + &c

As a test I started by looping through all the combinations 1 through 100 for each of the S1, S2and S3 values, looking for solutions near an integer like 5.000000000001 or 4.9999999999999 and calling it the integer value and checking it with 240 digits after the decimal point. Now by using your high school algebra to come up with a simple way to check and guess what it works. Now what is special about these values other than they come from the combinations of the three equations, see below.

S1 = ATANH(1/31) =lN(32/30)/2 = LN(16/15)/2
S2 = ATANH(1/49) =lN(50/48)/2 = LN(25/24)/2
S3 = ATANH(1/161) =lN(162/160)/2 = LN(81/80)/2

LN(N) = 2 * (K * LN(16/15)/2 + J * LN(25/24)/2 + L * LN(81/80)/2)
LN(N) = K * LN(16/15) + J * LN(25/24) + L * LN(81/80) clear out the 2's
LN(N) = LN((16/15)^K) + LN((25/24)^J) + LN((81/80)^L) convert the times log to powers

LN(N) = LN((16/15)^K * (25/24)^J * (81/80)^L) convert addition of log to times
N = (16/15)^K * (25/24)^J * (81/80)^L drop the logs as contents are equal
By using the values for LN(2)= 2 * (7 * S1 + 5 * S2 + 3 * S3)
So K=7, J=5 and L=3 and the equation becomes
2=(16/15)^7*(25/24)^5*(81/80)^3
2=(16*16*16*16*16*16*16*25*25*25*25*25*81*81*81)/(15*15*15*15*15*15*15*24*24*24*24*24*80*80*80)
2=(2^28*5^10*3^12)/(2^27*3^12*5^10)
2=2 it works just fine

Note that there are solutions for 2, 20, 200, 2000 and 20000 and the same is for 3, 4, 5 etc. There are no solutions for 7, 70, 700 and 7000. The only prime numbers are 2, 3 and 5 all the rest of the numbers are just composite of powers of those three numbers that is why they work. Even though I now know what is going on I did it is just for the fun of it. I have added at the end of each line the factoring of the log number. If you take the factoring and add the coefficients for each 2, 3 or 5 you will get the same numbers as in that line. For the line LN(4) you have 2*2 add the 7*S1+7S1=14*S1 which is the correct value and the same for the other two S2 and S3. What is interesting this is the order the program produced them I did not have to sort to get this final solution order.

LN(2) = 2*(7*S1 + 5*S2 + 3*S3) (2)= 2
LN(3) = 2*(11*S1 + 8*S2 + 5*S3) (3)= 3
LN(4) = 2*(14*S1 + 10*S2 + 6*S3) (4)= 2*2
LN(5) = 2*(16*S1 + 12*S2 + 7*S3) (5)= 5
LN(6) = 2*(18*S1 + 13*S2 + 8*S3) (6)= 2*3
LN(8) = 2*(21*S1 + 15*S2 + 9*S3) (8)= 2*2*2
LN(9) = 2*(22*S1 + 16*S2 + 10*S3) (9)= 3*3
LN(10) = 2*(23*S1 + 17*S2 + 10*S3) (10)= 2*5
LN(12) = 2*(25*S1 + 18*S2 + 11*S3) (12)= 2*2*3
LN(15) = 2*(27*S1 + 20*S2 + 12*S3) (15)= 3*5
LN(16) = 2*(28*S1 + 20*S2 + 12*S3) (16)= 2*2*2*2
LN(18) = 2*(29*S1 + 21*S2 + 13*S3) (18)= 2*3*3
LN(20) = 2*(30*S1 + 22*S2 + 13*S3) (20)= 2*2*5
LN(24) = 2*(32*S1 + 23*S2 + 14*S3) (24)= 2*2*2*3
LN(25) = 2*(32*S1 + 24*S2 + 14*S3) (25)= 5*5
LN(27) = 2*(33*S1 + 24*S2 + 15*S3) (27)= 3*3*3
LN(30) = 2*(34*S1 + 25*S2 + 15*S3) (30)= 2*3*5
LN(32) = 2*(35*S1 + 25*S2 + 15*S3) (32)= 2*2*2*2*2
LN(36) = 2*(36*S1 + 26*S2 + 16*S3) (36)= 2*2*3*3
LN(40) = 2*(37*S1 + 27*S2 + 16*S3) (40)= 2*2*2*5
LN(45) = 2*(38*S1 + 28*S2 + 17*S3) (45)= 3*3*5
LN(48) = 2*(39*S1 + 28*S2 + 17*S3) (48)= 2*2*2*2*3
LN(50) = 2*(39*S1 + 29*S2 + 17*S3) (50)= 2*5*5
LN(54) = 2*(40*S1 + 29*S2 + 18*S3) (54)= 2*3*3*3
LN(60) = 2*(41*S1 + 30*S2 + 18*S3) (60)= 2*2*3*5
LN(64) = 2*(42*S1 + 30*S2 + 18*S3) (64)= 2*2*2*2*2*2
LN(72) = 2*(43*S1 + 31*S2 + 19*S3) (72)= 2*2*2*3*3
LN(75) = 2*(43*S1 + 32*S2 + 19*S3) (75)= 3*5*5
LN(80) = 2*(44*S1 + 32*S2 + 19*S3) (80)= 2*2*2*2*5
LN(81) = 2*(44*S1 + 32*S2 + 20*S3) (81)= 3*3*3*3
LN(90) = 2*(45*S1 + 33*S2 + 20*S3) (90)= 2*3*3*5
LN(96) = 2*(46*S1 + 33*S2 + 20*S3) (96)= 2*2*2*2*2*3

LN(100) = 2*(46*S1 + 34*S2 + 20*S3) (100)= 2*2*5*5
LN(108) = 2*(47*S1 + 34*S2 + 21*S3) (108)= 2*2*3*3*3
LN(120) = 2*(48*S1 + 35*S2 + 21*S3) (120)= 2*2*2*3*5
LN(125) = 2*(48*S1 + 36*S2 + 21*S3) (125)= 5*5*5
LN(128) = 2*(49*S1 + 35*S2 + 21*S3) (128)= 2*2*2*2*2*2*2
LN(135) = 2*(49*S1 + 36*S2 + 22*S3) (135)= 3*3*3*5
LN(144) = 2*(50*S1 + 36*S2 + 22*S3) (144)= 2*2*2*2*3*3
LN(150) = 2*(50*S1 + 37*S2 + 22*S3) (150)= 2*3*5*5
LN(160) = 2*(51*S1 + 37*S2 + 22*S3) (160)= 2*2*2*2*2*5
LN(162) = 2*(51*S1 + 37*S2 + 23*S3) (162)= 2*3*3*3*3
LN(180) = 2*(52*S1 + 38*S2 + 23*S3) (180)= 2*2*3*3*5
LN(192) = 2*(53*S1 + 38*S2 + 23*S3) (192)= 2*2*2*2*2*2*3
LN(200) = 2*(53*S1 + 39*S2 + 23*S3) (200)= 2*2*2*5*5
LN(216) = 2*(54*S1 + 39*S2 + 24*S3) (216)= 2*2*2*3*3*3
LN(225) = 2*(54*S1 + 40*S2 + 24*S3) (225)= 3*3*5*5
LN(240) = 2*(55*S1 + 40*S2 + 24*S3) (240)= 2*2*2*2*3*5
LN(243) = 2*(55*S1 + 40*S2 + 25*S3) (243)= 3*3*3*3*3
LN(250) = 2*(55*S1 + 41*S2 + 24*S3) (250)= 2*5*5*5
LN(256) = 2*(56*S1 + 40*S2 + 24*S3) (256)= 2*2*2*2*2*2*2*2
LN(270) = 2*(56*S1 + 41*S2 + 25*S3) (270)= 2*3*3*3*5
LN(288) = 2*(57*S1 + 41*S2 + 25*S3) (288)= 2*2*2*2*2*3*3
LN(300) = 2*(57*S1 + 42*S2 + 25*S3) (300)= 2*2*3*5*5
LN(320) = 2*(58*S1 + 42*S2 + 25*S3) (320)= 2*2*2*2*2*2*5
LN(324) = 2*(58*S1 + 42*S2 + 26*S3) (324)= 2*2*3*3*3*3
LN(360) = 2*(59*S1 + 43*S2 + 26*S3) (360)= 2*2*2*3*3*5
LN(375) = 2*(59*S1 + 44*S2 + 26*S3) (375)= 3*5*5*5
LN(384) = 2*(60*S1 + 43*S2 + 26*S3) (384)= 2*2*2*2*2*2*2*3
LN(400) = 2*(60*S1 + 44*S2 + 26*S3) (400)= 2*2*2*2*5*5
LN(405) = 2*(60*S1 + 44*S2 + 27*S3) (405)= 3*3*3*3*5
LN(432) = 2*(61*S1 + 44*S2 + 27*S3) (432)= 2*2*2*2*3*3*3
LN(450) = 2*(61*S1 + 45*S2 + 27*S3) (450)= 2*3*3*5*5
LN(480) = 2*(62*S1 + 45*S2 + 27*S3) (480)= 2*2*2*2*2*3*5
LN(486) = 2*(62*S1 + 45*S2 + 28*S3) (486)= 2*3*3*3*3*3
LN(500) = 2*(62*S1 + 46*S2 + 27*S3) (500)= 2*2*5*5*5
LN(512) = 2*(63*S1 + 45*S2 + 27*S3) (512)= 2*2*2*2*2*2*2*2*2
LN(540) = 2*(63*S1 + 46*S2 + 28*S3) (540)= 2*2*3*3*3*5
LN(576) = 2*(64*S1 + 46*S2 + 28*S3) (576)= 2*2*2*2*2*2*3*3
LN(600) = 2*(64*S1 + 47*S2 + 28*S3) (600)= 2*2*2*3*5*5
LN(625) = 2*(64*S1 + 48*S2 + 28*S3) (625)= 5*5*5*5
LN(640) = 2*(65*S1 + 47*S2 + 28*S3) (640)= 2*2*2*2*2*2*2*5
LN(648) = 2*(65*S1 + 47*S2 + 29*S3) (648)= 2*2*2*3*3*3*3
LN(675) = 2*(65*S1 + 48*S2 + 29*S3) (675)= 3*3*3*5*5
LN(720) = 2*(66*S1 + 48*S2 + 29*S3) (720)= 2*2*2*2*3*3*5
LN(729) = 2*(66*S1 + 48*S2 + 30*S3) (729)= 3*3*3*3*3*3
LN(750) = 2*(66*S1 + 49*S2 + 29*S3) (750)= 2*3*5*5*5
LN(768) = 2*(67*S1 + 48*S2 + 29*S3) (768)= 2*2*2*2*2*2*2*2*3
LN(800) = 2*(67*S1 + 49*S2 + 29*S3) (800)= 2*2*2*2*2*5*5
LN(810) = 2*(67*S1 + 49*S2 + 30*S3) (810)= 2*3*3*3*3*5
LN(864) = 2*(68*S1 + 49*S2 + 30*S3) (864)= 2*2*2*2*2*3*3*3
LN(900) = 2*(68*S1 + 50*S2 + 30*S3) (900)= 2*2*3*3*5*5
LN(960) = 2*(69*S1 + 50*S2 + 30*S3) (960)= 2*2*2*2*2*2*3*5
LN(972) = 2*(69*S1 + 50*S2 + 31*S3) (972)= 2*2*3*3*3*3*3
LN(1000) = 2*(69*S1 + 51*S2 + 30*S3) (1000)= 2*2*2*5*5*5

LN(1024) = 2*(70*S1 + 50*S2 + 30*S3) (1024)= 2*2*2*2*2*2*2*2*2*2
LN(1080) = 2*(70*S1 + 51*S2 + 31*S3) (1080)= 2*2*2*3*3*3*5
LN(1125) = 2*(70*S1 + 52*S2 + 31*S3) (1125)= 3*3*5*5*5
LN(1152) = 2*(71*S1 + 51*S2 + 31*S3) (1152)= 2*2*2*2*2*2*2*3*3
LN(1200) = 2*(71*S1 + 52*S2 + 31*S3) (1200)= 2*2*2*2*3*5*5
LN(1215) = 2*(71*S1 + 52*S2 + 32*S3) (1215)= 3*3*3*3*3*5
LN(1250) = 2*(71*S1 + 53*S2 + 31*S3) (1250)= 2*5*5*5*5
LN(1280) = 2*(72*S1 + 52*S2 + 31*S3) (1280)= 2*2*2*2*2*2*2*2*5
LN(1296) = 2*(72*S1 + 52*S2 + 32*S3) (1296)= 2*2*2*2*3*3*3*3
LN(1350) = 2*(72*S1 + 53*S2 + 32*S3) (1350)= 2*3*3*3*5*5
LN(1440) = 2*(73*S1 + 53*S2 + 32*S3) (1440)= 2*2*2*2*2*3*3*5
LN(1458) = 2*(73*S1 + 53*S2 + 33*S3) (1458)= 2*3*3*3*3*3*3
LN(1500) = 2*(73*S1 + 54*S2 + 32*S3) (1500)= 2*2*3*5*5*5
LN(1536) = 2*(74*S1 + 53*S2 + 32*S3) (1536)= 2*2*2*2*2*2*2*2*2*3
LN(1600) = 2*(74*S1 + 54*S2 + 32*S3) (1600)= 2*2*2*2*2*2*5*5
LN(1620) = 2*(74*S1 + 54*S2 + 33*S3) (1620)= 2*2*3*3*3*3*5
LN(1728) = 2*(75*S1 + 54*S2 + 33*S3) (1728)= 2*2*2*2*2*2*3*3*3
LN(1800) = 2*(75*S1 + 55*S2 + 33*S3) (1800)= 2*2*2*3*3*5*5
LN(1875) = 2*(75*S1 + 56*S2 + 33*S3) (1875)= 3*5*5*5*5
LN(1920) = 2*(76*S1 + 55*S2 + 33*S3) (1920)= 2*2*2*2*2*2*2*3*5
LN(1944) = 2*(76*S1 + 55*S2 + 34*S3) (1944)= 2*2*2*3*3*3*3*3
LN(2000) = 2*(76*S1 + 56*S2 + 33*S3) (2000)= 2*2*2*2*5*5*5
LN(2025) = 2*(76*S1 + 56*S2 + 34*S3) (2025)= 3*3*3*3*5*5
LN(2048) = 2*(77*S1 + 55*S2 + 33*S3) (2048)= 2*2*2*2*2*2*2*2*2*2*2
LN(2160) = 2*(77*S1 + 56*S2 + 34*S3) (2160)= 2*2*2*2*3*3*3*5
LN(2187) = 2*(77*S1 + 56*S2 + 35*S3) (2187)= 3*3*3*3*3*3*3
LN(2250) = 2*(77*S1 + 57*S2 + 34*S3) (2250)= 2*3*3*5*5*5
LN(2304) = 2*(78*S1 + 56*S2 + 34*S3) (2304)= 2*2*2*2*2*2*2*2*3*3
LN(2400) = 2*(78*S1 + 57*S2 + 34*S3) (2400)= 2*2*2*2*2*3*5*5
LN(2430) = 2*(78*S1 + 57*S2 + 35*S3) (2430)= 2*3*3*3*3*3*5
LN(2500) = 2*(78*S1 + 58*S2 + 34*S3) (2500)= 2*2*5*5*5*5
LN(2560) = 2*(79*S1 + 57*S2 + 34*S3) (2560)= 2*2*2*2*2*2*2*2*2*5
LN(2592) = 2*(79*S1 + 57*S2 + 35*S3) (2592)= 2*2*2*2*2*3*3*3*3
LN(2700) = 2*(79*S1 + 58*S2 + 35*S3) (2700)= 2*2*3*3*3*5*5
LN(2880) = 2*(80*S1 + 58*S2 + 35*S3) (2880)= 2*2*2*2*2*2*3*3*5
LN(2916) = 2*(80*S1 + 58*S2 + 36*S3) (2916)= 2*2*3*3*3*3*3*3
LN(3000) = 2*(80*S1 + 59*S2 + 35*S3) (3000)= 2*2*2*3*5*5*5
LN(3125) = 2*(80*S1 + 60*S2 + 35*S3) (3125)= 5*5*5*5*5
LN(3072) = 2*(81*S1 + 58*S2 + 35*S3) (3072)= 2*2*2*2*2*2*2*2*2*2*3
LN(3200) = 2*(81*S1 + 59*S2 + 35*S3) (3200)= 2*2*2*2*2*2*2*5*5
LN(3240) = 2*(81*S1 + 59*S2 + 36*S3) (3240)= 2*2*2*3*3*3*3*5
LN(3375) = 2*(81*S1 + 60*S2 + 36*S3) (3375)= 3*3*3*5*5*5
LN(3456) = 2*(82*S1 + 59*S2 + 36*S3) (3456)= 2*2*2*2*2*2*2*3*3*3
LN(3600) = 2*(82*S1 + 60*S2 + 36*S3) (3600)= 2*2*2*2*3*3*5*5
LN(3645) = 2*(82*S1 + 60*S2 + 37*S3) (3645)= 3*3*3*3*3*3*5
LN(3750) = 2*(82*S1 + 61*S2 + 36*S3) (3750)= 2*3*5*5*5*5
LN(3840) = 2*(83*S1 + 60*S2 + 36*S3) (3840)= 2*2*2*2*2*2*2*2*3*5
LN(3888) = 2*(83*S1 + 60*S2 + 37*S3) (3888)= 2*2*2*2*3*3*3*3*3
LN(4000) = 2*(83*S1 + 61*S2 + 36*S3) (4000)= 2*2*2*2*2*5*5*5
LN(4050) = 2*(83*S1 + 61*S2 + 37*S3) (4050)= 2*3*3*3*3*5*5
LN(4096) = 2*(84*S1 + 60*S2 + 36*S3) (4096)= 2*2*2*2*2*2*2*2*2*2*2*2
LN(4320) = 2*(84*S1 + 61*S2 + 37*S3) (4320)= 2*2*2*2*2*3*3*3*5
LN(4374) = 2*(84*S1 + 61*S2 + 38*S3) (4374)= 2*3*3*3*3*3*3*3

LN(4500) = 2*(84*S1 + 62*S2 + 37*S3) (4500)= 2*2*3*3*5*5*5
LN(4608) = 2*(85*S1 + 61*S2 + 37*S3) (4608)= 2*2*2*2*2*2*2*2*2*3*3
LN(4800) = 2*(85*S1 + 62*S2 + 37*S3) (4800)= 2*2*2*2*2*2*3*5*5
LN(4860) = 2*(85*S1 + 62*S2 + 38*S3) (4860)= 2*2*3*3*3*3*3*5
LN(5000) = 2*(85*S1 + 63*S2 + 37*S3) (5000)= 2*2*2*5*5*5*5
LN(5120) = 2*(86*S1 + 62*S2 + 37*S3) (5120)= 2*2*2*2*2*2*2*2*2*2*5
LN(5184) = 2*(86*S1 + 62*S2 + 38*S3) (5184)= 2*2*2*2*2*2*3*3*3*3
LN(5400) = 2*(86*S1 + 63*S2 + 38*S3) (5400)= 2*2*2*3*3*3*5*5
LN(5625) = 2*(86*S1 + 64*S2 + 38*S3) (5625)= 3*3*5*5*5*5
LN(5760) = 2*(87*S1 + 63*S2 + 38*S3) (5760)= 2*2*2*2*2*2*2*3*3*5
LN(5832) = 2*(87*S1 + 63*S2 + 39*S3) (5832)= 2*2*2*3*3*3*3*3*3
LN(6000) = 2*(87*S1 + 64*S2 + 38*S3) (6000)= 2*2*2*2*3*5*5*5
LN(6075) = 2*(87*S1 + 64*S2 + 39*S3) (6075)= 3*3*3*3*3*5*5
LN(6250) = 2*(87*S1 + 65*S2 + 38*S3) (6250)= 2*5*5*5*5*5
LN(6144) = 2*(88*S1 + 63*S2 + 38*S3) (6144)= 2*2*2*2*2*2*2*2*2*2*2*3
LN(6400) = 2*(88*S1 + 64*S2 + 38*S3) (6400)= 2*2*2*2*2*2*2*2*5*5
LN(6480) = 2*(88*S1 + 64*S2 + 39*S3) (6480)= 2*2*2*2*3*3*3*3*5
LN(6561) = 2*(88*S1 + 64*S2 + 40*S3) (6561)= 3*3*3*3*3*3*3*3
LN(6750) = 2*(88*S1 + 65*S2 + 39*S3) (6750)= 2*3*3*3*5*5*5
LN(6912) = 2*(89*S1 + 64*S2 + 39*S3) (6912)= 2*2*2*2*2*2*2*2*3*3*3
LN(7200) = 2*(89*S1 + 65*S2 + 39*S3) (7200)= 2*2*2*2*2*3*3*5*5
LN(7290) = 2*(89*S1 + 65*S2 + 40*S3) (7290)= 2*3*3*3*3*3*3*5
LN(7500) = 2*(89*S1 + 66*S2 + 39*S3) (7500)= 2*2*3*5*5*5*5
LN(7680) = 2*(90*S1 + 65*S2 + 39*S3) (7680)= 2*2*2*2*2*2*2*2*2*3*5
LN(7776) = 2*(90*S1 + 65*S2 + 40*S3) (7776)= 2*2*2*2*2*3*3*3*3*3
LN(8000) = 2*(90*S1 + 66*S2 + 39*S3) (8000)= 2*2*2*2*2*2*5*5*5
LN(8100) = 2*(90*S1 + 66*S2 + 40*S3) (8100)= 2*2*3*3*3*3*5*5
LN(8192) = 2*(91*S1 + 65*S2 + 39*S3) (8192)= 2*2*2*2*2*2*2*2*2*2*2*2*2
LN(8640) = 2*(91*S1 + 66*S2 + 40*S3) (8640)= 2*2*2*2*2*2*3*3*3*5
LN(8748) = 2*(91*S1 + 66*S2 + 41*S3) (8748)= 2*2*3*3*3*3*3*3*3
LN(9000) = 2*(91*S1 + 67*S2 + 40*S3) (9000)= 2*2*2*3*3*5*5*5
LN(9375) = 2*(91*S1 + 68*S2 + 40*S3) (9375)= 3*5*5*5*5*5
LN(9216) = 2*(92*S1 + 66*S2 + 40*S3) (9216)= 2*2*2*2*2*2*2*2*2*2*3*3
LN(9600) = 2*(92*S1 + 67*S2 + 40*S3) (9600)= 2*2*2*2*2*2*2*3*5*5
LN(9720) = 2*(92*S1 + 67*S2 + 41*S3) (9720)= 2*2*2*3*3*3*3*3*5
LN(10000) = 2*(92*S1 + 68*S2 + 40*S3) (10000)= 2*2*2*2*5*5*5*5
LN(10125) = 2*(92*S1 + 68*S2 + 41*S3) (10125)= 3*3*3*3*5*5*5
LN(10240) = 2*(93*S1 + 67*S2 + 40*S3) (10240)= 2*2*2*2*2*2*2*2*2*2*2*5
LN(10368) = 2*(93*S1 + 67*S2 + 41*S3) (10368)= 2*2*2*2*2*2*2*3*3*3*3
LN(10800) = 2*(93*S1 + 68*S2 + 41*S3) (10800)= 2*2*2*2*3*3*3*5*5
LN(10935) = 2*(93*S1 + 68*S2 + 42*S3) (10935)= 3*3*3*3*3*3*3*5
LN(11250) = 2*(93*S1 + 69*S2 + 41*S3) (11250)= 2*3*3*5*5*5*5
LN(11520) = 2*(94*S1 + 68*S2 + 41*S3) (11520)= 2*2*2*2*2*2*2*2*3*3*5
LN(11664) = 2*(94*S1 + 68*S2 + 42*S3) (11664)= 2*2*2*2*3*3*3*3*3*3
LN(12000) = 2*(94*S1 + 69*S2 + 41*S3) (12000)= 2*2*2*2*2*3*5*5*5
LN(12150) = 2*(94*S1 + 69*S2 + 42*S3) (12150)= 2*3*3*3*3*3*5*5
LN(12500) = 2*(94*S1 + 70*S2 + 41*S3) (12500)= 2*2*5*5*5*5*5
LN(12288) = 2*(95*S1 + 68*S2 + 41*S3) (12288)= 2*2*2*2*2*2*2*2*2*2*2*2*3
LN(12800) = 2*(95*S1 + 69*S2 + 41*S3) (12800)= 2*2*2*2*2*2*2*2*2*5*5
LN(12960) = 2*(95*S1 + 69*S2 + 42*S3) (12960)= 2*2*2*2*2*3*3*3*3*5
LN(13122) = 2*(95*S1 + 69*S2 + 43*S3) (13122)= 2*3*3*3*3*3*3*3*3
LN(13500) = 2*(95*S1 + 70*S2 + 42*S3) (13500)= 2*2*3*3*3*5*5*5
LN(13824) = 2*(96*S1 + 69*S2 + 42*S3) (13824)= 2*2*2*2*2*2*2*2*2*3*3*3

LN(14400) = 2*(96*S1 + 70*S2 + 42*S3) (14400)= 2*2*2*2*2*2*3*3*5*5
LN(14580) = 2*(96*S1 + 70*S2 + 43*S3) (14580)= 2*2*3*3*3*3*3*3*5
LN(15000) = 2*(96*S1 + 71*S2 + 42*S3) (15000)= 2*2*2*3*5*5*5*5
LN(15625) = 2*(96*S1 + 72*S2 + 42*S3) (15625)= 5*5*5*5*5*5
LN(15360) = 2*(97*S1 + 70*S2 + 42*S3) (15360)= 2*2*2*2*2*2*2*2*2*2*3*5
LN(15552) = 2*(97*S1 + 70*S2 + 43*S3) (15552)= 2*2*2*2*2*2*3*3*3*3*3
LN(16000) = 2*(97*S1 + 71*S2 + 42*S3) (16000)= 2*2*2*2*2*2*2*5*5*5
LN(16200) = 2*(97*S1 + 71*S2 + 43*S3) (16200)= 2*2*2*3*3*3*3*5*5
LN(16875) = 2*(97*S1 + 72*S2 + 43*S3) (16875)= 3*3*3*5*5*5*5
LN(16384) = 2*(98*S1 + 70*S2 + 42*S3) (16384)= 2*2*2*2*2*2*2*2*2*2*2*2*2*2
LN(17280) = 2*(98*S1 + 71*S2 + 43*S3) (17280)= 2*2*2*2*2*2*2*3*3*3*5
LN(17496) = 2*(98*S1 + 71*S2 + 44*S3) (17496)= 2*2*2*3*3*3*3*3*3*3
LN(18000) = 2*(98*S1 + 72*S2 + 43*S3) (18000)= 2*2*2*2*3*3*5*5*5
LN(18225) = 2*(98*S1 + 72*S2 + 44*S3) (18225)= 3*3*3*3*3*3*5*5
LN(18750) = 2*(98*S1 + 73*S2 + 43*S3) (18750)= 2*3*5*5*5*5*5
LN(18432) = 2*(99*S1 + 71*S2 + 43*S3) (18432)= 2*2*2*2*2*2*2*2*2*2*2*3*3
LN(19200) = 2*(99*S1 + 72*S2 + 43*S3) (19200)= 2*2*2*2*2*2*2*2*3*5*5
LN(19440) = 2*(99*S1 + 72*S2 + 44*S3) (19440)= 2*2*2*2*3*3*3*3*3*5
LN(19683) = 2*(99*S1 + 72*S2 + 45*S3) (19683)= 3*3*3*3*3*3*3*3*3
LN(20000) = 2*(99*S1 + 73*S2 + 43*S3) (20000)= 2*2*2*2*2*5*5*5*5
LN(20250) = 2*(99*S1 + 73*S2 + 44*S3) (20250)= 2*3*3*3*3*5*5*5
LN(20480) = 2*(100*S1 + 72*S2 + 43*S3) (20480)= 2*2*2*2*2*2*2*2*2*2*2*2*5
LN(20736) = 2*(100*S1 + 72*S2 + 44*S3) (20736)= 2*2*2*2*2*2*2*2*3*3*3*3
LN(21600) = 2*(100*S1 + 73*S2 + 44*S3) (21600)= 2*2*2*2*2*3*3*3*5*5
LN(21870) = 2*(100*S1 + 73*S2 + 45*S3) (21870)= 2*3*3*3*3*3*3*3*5
LN(22500) = 2*(100*S1 + 74*S2 + 44*S3) (22500)= 2*2*3*3*5*5*5*5

More of William Shanks other work starting with the work at "archive.org/stream/philtrans04897979/04897979#page/n0/mode/1up" Here is another calculation of another important Math number.

Fourth and concluding Supplementary Paper on the Calculation of the Numerical Value of Euler's Constant by William Shanks. Communicated by Professor Stokes, Sec. R.S. Received June 14, 1869.

when n=10000 we have 1+1/2+1/3+…+1/10000=
This is my calculations to 105 digits rounded down to 90 digits.
9.78760 60360 44382 26417 84779 04851 60533 48592 62945 57769
17183 89460 95668 16020 24943 15950 68001 25127

William Shanks value this looks like the normal error due to all the 10000 calculations.
9.78760 60360 44382 26417 84779 04851 60533 48592 62945 577**72** a typo error.
17183 89460 9**7673 221+**

LN(10000)+1/20000
This is my calculations to 105 digits rounded down to 90 digits. why he added 1/20000 I am not sure.
9.21039 03719 76182 73607 19658 18737 45683 04044 05954 51509
19041 33311 60387 02904 38709 40992 09439 88820

William Shanks value looks like the normal possible error depending how the calculations were done. Remember he has to do 10,000 divisions and additions.

This is my calculations to 105 digits rounded down to 90 digits. Why he added 1/20000 I am not sure.
9.21039 03719 76182 73607 19658 18737 45683 04044 05954 51509
19041 333**11 60387 029**

E= Euler's Constant
This is my calculations to 105 digits rounded down to 90 digits.

.57721 56649 01532 86060 65120 90082 40243 10421 59335 93992
35988 05767 23488 48677 26777 66467 09369 47063

William Shanks value this looks like the normal possible error depending how the calculations were done.
.57721 56649 01532 86060 65120 90082 40243 10421 59335 9399**5** I think this digit was a typo error.
35988 057**67 23488 486**

Second Supplementary Paper on the Calculation of the Numerical Value of Euler's Constant by William Shanks. Communicated by Professor Stokes, Sec. R.S. Received August 29, 1867. Using only 2000 terms and he obtained the following values. This paper can be found at "archive.org/stream/philtrans04947669/04947669#page/n0/mode/2up"

This is my calculations to 120 digits rounded down to 105 digits.
8.17836 81036 10282 40957 76565 71641 69368 79354
66740 91248 77402 20409 26320 14205 58139 78438
87946 27554 87331 13641 80365

William Shanks value this looks like the normal possible error depending how the calculations were done.
8.17836 81036 10282 40957 76565 71641 69368 79354
66740 912**51** 77402 20409 26320 14205 58139 78438 another typo error
87946 27554 87**631 87631 13634 +**

This is my calculations to 120 digits rounded down to 75 digits.
0.57721 56649 01532 86060 65120 90082 40243 10421 59335
93992 35988 05767 23488 48677 26777 66467

William Shanks value this looks like the normal possible error depending how the calculations were done.
0.57721 56649 01532 86060 65120 90082 40243 10421 59335

9399**5** 35988 057**67 23488 48677 26777 66467** The same value as in 1969

Another calculation William Shanks worked on was the number of digits in the reciprocal of a prime number published on December 2, 1873 “Reciprocal of ever Prime Number below 20,000” can be found at “archive.org/details/philtrans04514494”. It is not known if he found a list of prime numbers or he also calculated the prime number list also. As an example 3 has 1 digit while 7 has 6 digits all the way up to 19997 which has 9998 digits. By June 6, 1874 he had completed up through 30, 000 see his results at “archive.org/details/philtrans09911316”. By January 5, 1875 He expanded his work to 40,000 and eventually ended at 60,000 but his last group was not published rather just mentioned just kept on record for any one to review.

As you can see he did work on other calculation other than just PI. He really must have been good at arithmetic for his enduring ability to do so much number calculation. I had a hard time to do 100 digits of PI by hand, I guess he did not spend as much time watching TV or working on the computer as I do and not working under electric lights. You can view my work at “engert.us/Erwin/miscellaneous/Calculating pi by hand for 100 digits.pdf” this file only shows 14 of the 108 hand calculation.

I do not expect to find another person like William Shanks who can do large number of hand calculation. After all it took 73 years before someone took up the job to check his work and find the error. They used a desk calculators and not doing a hand calculation. When I was in seventh grade my math teach brought in a mental calculating person who could add a large column of eight digit numbers by just going down the column once and writing the total answer at the bottom. He showed us other way he could do math solutions with very little effort and produced the correct answers. After his presentation I asked him if he is still as fast as he was years before and he admitted he was finding himself using a calculator and his speed was decreasing with time, he was afraid he would lose his ability with time and have to use a calculator for all his work. Today kids start out using a calculator and never perfect their ability to do mental math either in the mind or with paper and pencil only. See my challenge on page 9 of my file at “engert.us/Erwin/ miscellaneous/William Shanks 707 digits.pdf” to calculate PI to 40 digits by hand with paper and pencil only. Today I am beginning to think that we have past the point when someone can meet my challenge.

William Shanks did a lot of hand calculations although he did have errors; his total work was very remarkable by any standard. Today we are too stuck on having a machine do everything for us. If want to add two numbers we must pull out a cell phone and do any arithmetic we can no longer do it in our head. A friend of mine if he cannot find a calculator he does the math in using paper and pencil as he has done for decades. I have placed on my web site the long division for calculations of PI to 20, 40 and 100 digits see them at “engert.us/erwin/miscellaneous.html, for the fun of it I have also done 10, 15 and 30 digits not on the web site. I asked an 18 year old kid to tell me the total repeating digits for 1/19 and he pulled out his cell phone and gave me the eight digit that his phone. As it turns out there are 18 digits to the repeating pattern, well I told him he was wrong. I have done it in ten minutes by hand with just paper and pencil.